

For a homogeneous nuclear reactor producing a constant power, the following holds:

$$\frac{d}{dt} \log P = -\frac{\alpha}{\tau} T$$

$$\frac{d}{dt} T = (P-1) \frac{1}{\epsilon}$$

where

$$P(t) = \text{instantaneous reactor power}$$

$$-\alpha = \text{temperature coefficient}$$

$$\tau = \text{half time of a neutron}$$

$$\epsilon = \text{heat capacity}$$

To obtain the canonical form, use the transformation

$$x_1 = \log P \text{ and } x_2 = -\frac{\alpha}{\tau} T$$

This leads to the system

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{\alpha}{\epsilon \tau} (e^{x_1} - 1)$$

The Lyapunov function of this system is

$$V(\mathbf{x}) = \frac{1}{2} x_2^2 + \frac{\alpha}{\epsilon \tau} (e^{x_1} - x_1) - \frac{\alpha}{\epsilon \tau}$$

$$\dot{V}(\mathbf{x}) = -\left\{\frac{\alpha}{\tau}\right\}^2 \frac{T}{\epsilon} (P-1) < 0 \text{ if } P > 1 \text{ and } \dot{V} > 0 \text{ if } P < 1$$

$V(\mathbf{x})$ is positive definite so the equilibrium point $P = 1, T = 0$ is stable as the three parameters α, τ and ϵ are positive.

If P is less than 1, the reactor is unstable. Cooling is essential.

A well known fact.